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LOW CYCLE FATIGUE LIFE ESTIMATION FOR DUCTILE METALS USING A NONLINEAR CONTINUUM DAMAGE MECHANICS MODEL

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Abstract—Continuum damage mechanics is an effective approach to model ductile failure. The same concepts can be extended to the low cycle fatigue damage process, where plasticity is still the key mechanism for crack initiation. In addition, in low cycle fatigue a relevant part of life is spent by the material to initiate a macroscopic crack that leads to complete failure. In this paper, the nonlinear damage model, initially proposed by Bonora, N. (1997) A non-linear CDM model for ductile fracture *Engineering Fracture Mechanics* (in press), is extended to the case of cyclic loading. Three possible formulations are proposed and discussed that take into account in different ways the accumulation of damage, plastic strain and the material cyclic properties change. Fully coupled life model was used to predict low cycle fatigue life in Al 2024 T3 alloy and HY80 low carbon steel. Comparison with a large fatigue experimental data set is also presented. © 1998 Elsevier Science Ltd.

INTRODUCTION

It is well known that microvoids are one of the basic mechanisms for ductile failure process. The early studies of McClintock (1968) and Rice and Tracey (1969) clarified the key role of the microvoids that are generated as a result of debonding of included particles or second phase precipitates as carbides. These studies were also the first step to develop a consistent constitutive model to predict the possible evolution of voids in a ductile matrix. Later, Gurson (1977) developed a model for porous material to estimate the effect of cavities on the overall constitutive response of ductile materials. Recently, the Gurson's model has been modified by Needleman and Tvergaard (1984) to take into account the growth rate increase as a result of the coalescence process that takes place between voids prior to failure. Lemaitre (1985) and Lemaitre and Chaboche (1990) developed a formulation for damaged elastic–viscoplastic materials where damage is addressed as a continuum constitutive variable. In this framework, damage may be considered as a progressive material deterioration as a result of the irreversible deformation processes in the material micro-structure due to void formation and growth, local micro-cracking and their mutual interaction. The nonlinear behavior of the relation between damage and plasticity was investigated through the experimental work of Le Roy *et al.* (1981).

After this work, several nonlinear damage models, following the same scheme proposed by Lemaitre, have been proposed (Tai, 1990; Chandrakanth and Pandey, 1993). However, in most cases these models were able to describe only damage evolution for particular metals. Bonora (1997), recently proposed a nonlinear damage model capable of predicting different damage evolution for a wide range of ductile metals.

Because damage mechanics deals with damage in terms of continuum variable, it is particularly well suited to approach crack initiation. More specifically, CDM model for damage and plasticity can be used to predict ductile crack initiation and propagation up to the formation of a macroscopic flaw big enough to be subsequently described with elastic–

plastic fracture mechanics parameters such as J, or J_M (Ernst, 1983), together with a crack tip constraint parameter such as T-stress (Shih and German, 1981) or Q-stress (Betagòn and Hancock, 1991). The same approach may be used to predict fatigue behavior of metals when loads or imposed strain are high enough to induce plasticity in the material microstructure and where plasticity continues to play a fundamental role in the failure process. In this paper the nonlinear damage model proposed by Bonora (1997) is summarized, and its extension to the fatigue loading is proposed. The model is particularly well suited to describe low cycle fatigue process in ductile metal, where most of the fatigue life is spent by the material in developing a ductile crack, so that the crack growth part of life can be neglected. The model was verified comparing the predicted fatigue lives, for an aluminum alloy and a low carbon steel, with a large number of low cycle fatigue experimental data available in the literature (Boller and Seeger, 1987a, 1987b).

DAMAGE MODEL

Damage in continuum damage mechanics (CDM) is one of the constitutive variables that takes into account the degradation and loss of performance of materials that results in a stiffness reduction. Damage variable can be also related to the physical modifications in the material such as initiation and growth of microvoids and microcracks. In fact, if we consider an unitary reference volume element, the presence of damage reduces the effective net resisting area:

$$D = 1 - \frac{A_{\mathrm{eff}}}{A_0} \tag{1}$$

where A_0 is the nominal section area of a reference volume element dV, and A_{eff} the effective resisting section area reduced by damage. In eqn (1), damage variable can be defined using a scalar D assuming that damage is isotropic in the material microstructure and that plasticity or developing damage does not induce any anisotropy. This hypothesis can be taken as valid for a large range of applications. However, in the event of anisoptropic damage, tensorial damage formulation can be found in Murakami (1982) and Chaboche (1979). Using the Kachanov's definition of effective stress, it is possible to relate damage variable D to the reduction of stiffness as:

$$D = 1 - \frac{E_{\mathrm{eff}}}{E_0} \tag{2}$$

where E_0 is the initial Young's modulus and E_{eff} is the effective one.

Following the same approach proposed by Lemaitre (1985), in order to write the complete set of equations for a damaged material the following hypothesis are made:

(a) The existence of a damage dissipation potential F_{D}, similar to the one used for plasticity, is postulated.

(b) No coupling between damage and plasticity dissipation potentials is assumed. Then, the total dissipation potential can be given as:

$$F = F_{\mathrm{p}}(\sigma, R, X; D) + F_{\mathrm{D}}(Y; p, D) \tag{3}$$

where F_{p} is the dissipation potential associated to plastic deformation function of the actual stress tensor, σ, and of the isotropic and kinematic hardening back stress, R and X, respectively; F_{D} is the dissipation potential associated to the damage process where Y is the internal variable associated to damage and p the accumulated effective plastic strain.

(c) Damage variable, D, and plastic strain are coupled. As a matter of fact, plasticity damage is always related to the irreversible strain at microlevel or mesolevel (Lemaitre, 1992): in the kinetic law of damage evolution this property is expressed by the plastic

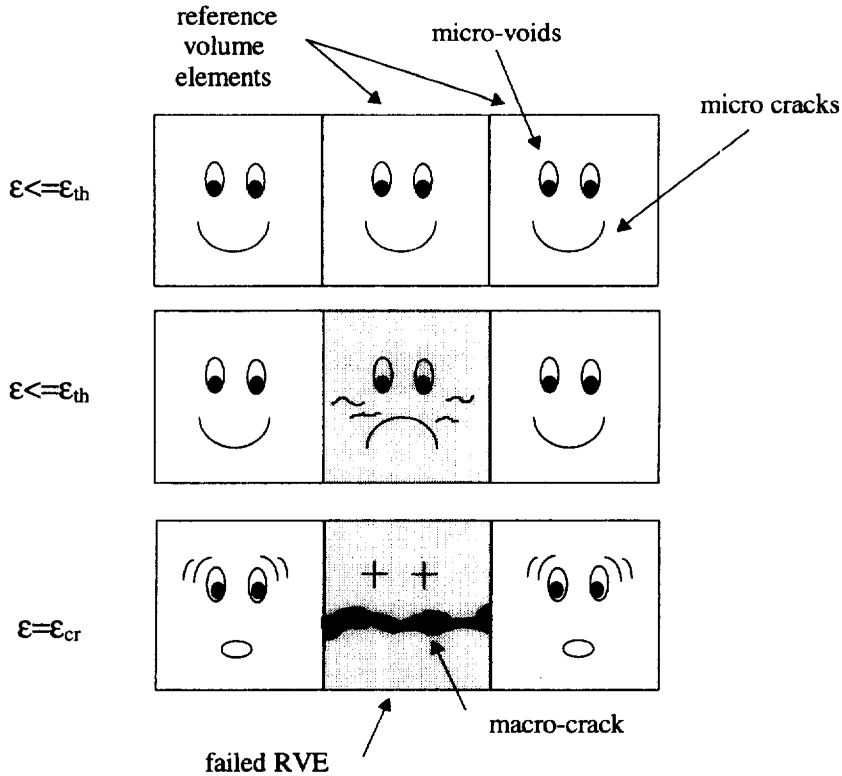

Fig. 1. Sketch of the effect of localized damage on material macrostructure.

multiplier λ which is proportional to the accumulated plastic strain p. Furthermore, plastic damage dissipation is associated to the micro-cavities growth that is well known to be a highly nonlinear process. For this reason the expression of the damage potential has to depend on the effective accumulated plastic strain that is an indicator of which growth phase microvoids are experiencing.

(d) Damage phenomena are localized in the material micro-scale and damage effects remain confined until the complete failure of several elementary volume elements, with the consequent appearance of a macroscopic crack, which occurs as shown in Fig. 1. The damage effect localization can be experimentally observed comparing the measured loss of stiffness in hour-glass specimens. This is obtained with a small strain gauge positioned onto the minimum section, where plastic deformation starts to develop also, a clip gauge centered across that section. The strain gauge measures material stiffness variation since the damage plastic strain threshold is exceeded; on the contrary, the clip gauge does not reveal any change in the Young's modulus until a macroscopic crack is formed (Bonora *et al.*, 1996b; Hancock and Mackenzie, 1976).

(e) Damage affects only stresses; total strain is the same in both macro and micro-scale (Taylor, 1938).

(f) Damaged material can be described using the same set of constitutive equations of the virgin material (used to describe the material behavior at macro-scale), where the stress is substituted by the effective stress and a state equation for damage variable has to be given.

In this framework it is possible to write the following constitutive equations set. Total strain decomposition in elastic and plastic contribution, is assumed. For elastic strain we get

$$\varepsilon_{ij}^{e} = \frac{1+\nu}{E}\frac{\sigma_{ij}}{1-D} - \frac{\nu}{E}\frac{\sigma_{kk}}{1-D}\delta_{ij}. \tag{4}$$

Standard isotropic plasticity associated with a Von Mises yield criterion leads to

$$F_{\mathrm{p}}(\sigma, R, X; D) = \left(\frac{s}{1-D} - X'\right)_{\mathrm{eq}} - R(r) - \frac{3}{4X_{\infty}} X'_{ij} X'_{ij} - \sigma_y \tag{5}$$

where s_{ij} and X'_{ij} are the deviatoric of the stress and kinematic hardening tensor, respectively; σ_y is the initial uniaxial yield stress, and

$$\left(\frac{s}{1-D} - X'\right)_{\mathrm{eq}} = \left[\frac{3}{2}\left(\frac{S_{ij}}{1-D} - X'_{ij}\right)\left(\frac{S_{ij}}{1-D} - X'_{ij}\right)\right]^{1/2}. \tag{6}$$

The plastic strain components, taking into account of the kinematic hardening, can be obtained as

$$\dot{\varepsilon}^{\mathrm{p}}_{ij} = \dot{\lambda}\frac{\partial F_{\mathrm{p}}}{\partial \sigma_{ij}} = \frac{3}{2}\frac{\dot{\lambda}}{1-D}\frac{\left(\frac{S_{ij}}{1-D} - X'_{ij}\right)}{\left(\frac{s}{1-D} - X'\right)_{\mathrm{eq}}} \tag{7}$$

$$\dot{\alpha}_{ij} = -\dot{\lambda}\frac{\partial F_{\mathrm{p}}}{\partial X'_{ij}} = \dot{\varepsilon}^{\mathrm{p}}_{ij}(1-D) - \frac{3}{2X_{\infty}} X'_{ij}\dot{\lambda} \tag{8}$$

$$\dot{r} = -\dot{\lambda}\frac{\partial F_{\mathrm{p}}}{\partial R} = \dot{\lambda} = \dot{p}(1-D) \tag{9}$$

where λ is the plastic multiplier.

The kinematic back stress increases with the plastic strain which is nonlinear with plastic strain and tends to saturate to a value, characteristic for the material, X_{∞}. More detailed description on the kinematic back stress formulation can be found elsewhere (Lemaitre, 1992). The kinetic law of damage evolution is then given by:

$$\dot{D} = -\dot{\lambda}\frac{\partial F_{\mathrm{D}}}{\partial Y}. \tag{10}$$

Bonora (1997) proposed the following expression for the damage dissipation potential

$$F_{\mathrm{D}} = \left[\frac{1}{2}\left(-\frac{Y}{S_0}\right)^2 \cdot \frac{S_0}{1-D}\right] \cdot \frac{(D_{\mathrm{cr}} - D)^{(\alpha-1)/\alpha}}{p^{(2+n)/n}} \tag{11}$$

where D_{cr} is the critical value of the damage variable for which ductile failure occurs; α is the damage exponent that characterize the shape of the damage evolution curve and it is strictly related to the nature of the bound between brittle inclusions and ductile matrix. S_0 is a material constant and n is the material hardening exponent.

Basically, the square bracketed term in eqn (11) is the classical quadratic form for the damage dissipation potential as initially proposed by Lemaitre. However, as a result of the fact that the damage dissipation potential is related to the microvoid growth process, it has to result in a different dissipation for a void that is nucleating, or is under steady growth or is coalescing with other voids.

Tvergaard and Needleman (1984) modified the porosity model initially proposed by Gurson introducing a two-slopes porosity function in order to simulate a more rapid decrease in the material stiffness as a consequence of voids coalescence. The identification of the exact plastic strain value for which each growth phase starts, or ends, is very difficult. In addition, it is not know *a priori* for a given reference volume element, how many voids are nucleating, how many are growing and, finally, how many are coalescing, and more than ever which is their mutual effect. In the model proposed by Bonora (1997), the term out of the square brackets in eqn (11) describing that ductile damage dissipation depends on the effective accumulated plastic strain level reached in the RVE. The damage exponent α describes (from the phenomenological point of view) that the way the three-phase void growth process evolves is under plastic deformation.

Using eqn (10), assuming for the material hardening a Ramberg–Osgood form, the kinetic damage evolution law leads to

$$dD = \alpha \cdot \frac{(D_{cr} - D_0)^{1/\alpha}}{\ln(\varepsilon_{cr}/\varepsilon_{th})} \cdot f\left(\frac{\sigma_H}{\sigma_{eq}}\right) \cdot (D_{cr} - D)^{(\alpha-1)/\alpha} \cdot \frac{dp}{p} \tag{12}$$

where D_0 is the initial damage in the material microstructure due to the presence of inclusions or second phase precipitates; and ε_{th} and ε_{cr} are the threshold strain at which damage process initiates and the strain at failure in the uniaxial state of stress, respectively. The function $f(\sigma_H/\sigma_{eq})$ takes into account the effect of stress triaxiality on the void growth process and is given by:

$$f\left(\frac{\sigma_H}{\sigma_{eq}}\right) = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2 \tag{13}$$

where ν is the Poisson's ratio and $\sigma_H(=1/3\sigma_{kk})$ is the trace of the stress tensor.

Detailed derivation of the above equations is given in Lemaitre (1992) and Bonora (1997). The model has been successfully verified on a large number of metals that exhibit a different damage evolution as a function of the plastic strain (Bonora, 1997). Damage evolution seems to be strongly dependent on the kind of interface between the ductile matrix and brittle particles, on its strength and on the matrix yield stress. When the interface strength is lower than the matrix yield stress, particles easily debond resulting in a rapid damage accumulation process. Therefore, once the damage is saturated, it produces failure in the material. If the yield stress is lower than the interface strength, only few particles will debond and, consequently, damage accumulates with a lower rate.

In Fig. 2, the comparison, between the experimental data and the model proposed by Bonora (1997), for three possible different "shapes" of the damage vs. plastic strain curves

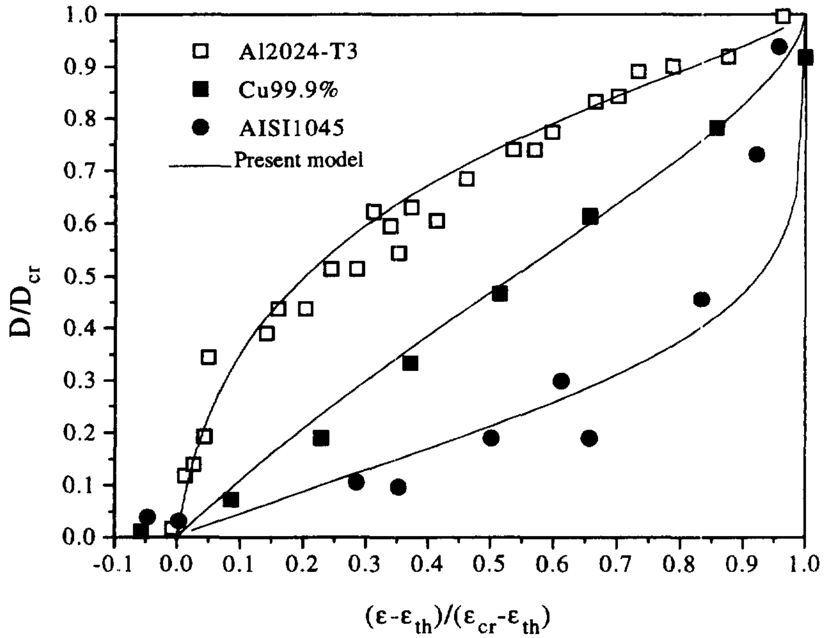

Fig. 2. Damage evolution vs strain: comparison between the damage model proposed by Bonora and experimental data relative to Cu99%, Al 2024-T3 and AISI 1045 low carbon steel.

Table 1. Summary of damage parameters

Material	ε_{th}	ε_{cr}	D_0	D_{cr}	α
Al 2024-T3	0.009	0.33	0.0	0.118	0.2136
Cu 99.9%	0.34	1.04	0.0	0.85	0.2
AISI 1045	0.223	0.93	0.0	0.065	0.2173

is given. Experimental damage measurements refer to an AL 2024-T3 alloy (Chow and Yang, 1987), pure Cu 99.9% (Lemaitre and Dufailly, 1977) and low carbon steel AISI 1045 (Le Roy *et al.*, 1981). Data in Fig. 2 have been obtained measuring damage from monotonic tensile tests as the reduction of the Young's modulus. Damage parameters, such as D_{cr}, ε_{th}, ε_f and α, have been determined according to the identification procedure given in detail by Bonora (1997).

In order to allow a comparison between the three different curves both strain and damage have been normalized respect to the relative critical and threshold value. This representation of data does not affect the real shape of curves. In Table 1 the damage material parameters for the three metals are summarized.

EXTENSION TO CYCLIC LOADING

It is well known that the application of cyclic loading activates the dislocation motion that sooner or later will lead to the formation of a fatigue crack. At the same time, cyclic loading can produce plastic deformations in the material microstructure as a result of the presence of "stress-raisers", as inclusions or pre-existing microvoids and flaws, or simply because the applied stress amplitude is strong enough to induce permanent strains.

In fatigue, a great distinction exists between the cyclic behavior of plain or notched specimen or component. Material behavior can be described by the use of the Wohler's diagram for high cycle fatigue, and Manson and Coffin's curve for low cycle fatigue. In both the cases, part of the fatigue life before complete failure is used by the material to generate a macroscopic crack. This phase can be a relevant portion of the entire fatigue life in the deformation process as for low cycle fatigue where the strain reversals can be so intense to produce material failure after few cycles after the appearance of a macro-crack. Thus, CDM can be used to follow the damaging process, as a result of cyclic plastic straining, up to the time of appearance of a crack long enough to be followed by traditional fracture mechanics approaches.

Let us consider a cyclic strain controlled loading where the total strain amplitude is constant

$$\Delta\varepsilon^{T} = \varepsilon_a^{T} = \varepsilon_a^{e} + \varepsilon_a^{p} = \text{const} \tag{14}$$

for this type of deformation process, stress amplitude can change sensibly due to material softening or hardening. For example, Morrow (1965) showed that the change in stress amplitude for a strain controlled test on fully annealed, 33% cold worked and 5% partially annealed copper spread from 400 to 33 and 15%, respectively. Anyway, these variations are stabilized or completed within 10–30% of the total fatigue life (Fuchs and Stephens, 1980). According to this we can assume that once Bauschinger's effects are saturated, the stress amplitude remains constant, then we can write:

$$\left.\begin{aligned} \varepsilon_a^{T} &= \text{const} \\ \sigma_a &= \text{const} \end{aligned}\right\} \rightarrow \varepsilon_a^{p} = \text{const} \tag{15}$$

then eqn (14) can be written as

$$\varepsilon_a^T = \frac{\sigma_a}{\tilde{E}} + \varepsilon_a^p \tag{16}$$

where $\tilde{E}$ is the effective material Young's modulus and σ_a is the stress amplitude defined as the half stress range: $\sigma_a = (\sigma_{max} - \sigma_{min})/2$. The plastic strain amplitude ε_a^p can be given in terms of stress amplitude through cyclic Ramberg–Osgood curve:

$$\varepsilon_a^p = \left(\frac{\sigma_{max}}{K'}\right)^{1/n'} \tag{17}$$

where K' and n' are constants available in handbooks for many materials. σ_{max} is the maximum applied stress that in cyclic response test is equivalent to σ_a for zero mean stress, $\sigma_m = 0$. During cycling, damage processes are not active until the total accumulated plastic strain is equal or higher than threshold strain, p_{th}. This means that the number of cycles necessary for the failure of some reference volume elements, i.e. the formation of a macro-crack, can be given as the sum of two contributions: N_{th} cycles necessary to cumulate p_{th} strain (during this cycles damage processes are not active) and N_D cycles necessary to reach the critical damage D_{cr}

$$N_f = N_{th} + N_D. \tag{18}$$

If we make the hypothesis that the material behaves in the same way both in tension and in compression, the first contribution of eqn (18) can be easily calculated taking into account that during each total strain reversal, the plastic strain accumulates to an amount equal to ε_a^p. Then, the number of cycles necessary to accumulate plastic deformation equal to p_{th} is given by:

$$N_{th} = \frac{p_{th}}{\varepsilon_a^p}. \tag{19}$$

When the accumulated plastic strain p reaches the strain threshold, damage processes are activated and damage variable starts to accumulate at each cycle (or strain reversal). The accumulation process proceeds until damage variable reaches the critical value D_{cr}. For a generic load cycle, in the case of proportional loading $f(\sigma_H/\sigma_{eq}) = 1$, the amount of damage produced can be calculated integrating the damage law given in eqn (12) as follows:

$$\left(\frac{\partial D}{\partial N}\right)_{N=N_i} = \int_{D_{i-1}}^{D} \frac{\mathrm{d}D}{(D_{cr} - D)^{(\alpha-1)/\alpha}} = \alpha \frac{(D_{cr} - D_0)^{1/\alpha}}{\ln(\varepsilon_{cr}/\varepsilon_{th})} \cdot \int_{p_{i-1}}^{p_{i-1}-\varepsilon_a^p} \frac{\mathrm{d}p}{p} \tag{20}$$

where N_i is the actual cycle, D_{i-1} and P_{i-1} are the amount of damage and the plastic strain accumulated in the previous cycles, respectively. If the evolution law of plastic strain during the single cycle is neglected, we get:

$$\left(\frac{\partial D}{\partial N}\right)_{N=N_i} = D_{i-1} + (D_{cr} - D_{i-1}) \cdot \left\{1 - \left[1 - \left(\frac{D_{cr} - D_0}{D_{cr} - D_{i-1}}\right)^{1/\alpha} \frac{\ln[(p_{i-1} + \varepsilon_a^p)/p_{i-1}]}{\ln(\varepsilon_{cr}/\varepsilon_{th})}\right]^{\alpha}\right\}. \tag{21}$$

For example if the applied plastic strain amplitude is equal to $(\varepsilon_{cr} - \varepsilon_{th})$, the number of cycles N_{th} is zero (less than 1!) and the amount of damage produced in the first damaging cycle is equal to D_{cr}: this case of the uniaxial loading up to failure strain. On the other hand, if the imposed plastic strain amplitude is zero, the amount of damage produced per cycle is also zero and, consequently, life, intended as the number of cycles necessary to produce a crack, is infinite. In the other cases, to estimate the fatigue life for which a macro crack is initiated in the material, it is sufficient to sum the single damage contribution per cycle until D reaches D_{cr}.

Because the amount of damage produced during a specific cycle is function of the past strain history, it is possible to think to estimate low cycle fatigue life according to three different coupling levels between cumulated damage, cumulated plastic strain and cycles.

Uncoupled life model

Low cycle fatigue life can be easily calculated if the plastic strain accumulation process is not taken into account during the damaging cycles, N_D. It is possible to assume that each cycle produces the same amount of damage as for the first damaging cycle. In this case, it is possible to integrate directly the amount of damage per cycle given in eqn (21) between the $N = N_{th}$ and $N = N_f$ for which $D = D_{cr}$. This leads to:

$$\int_{D_0}^{D_{cr}} dD = \int_{N_{th}}^{N_f} \left(\frac{\partial D}{\partial N}\right) dN. \tag{22}$$

Imposing for the first damaging cycle $p_{i-1} = p_{th}$ and $D_{i-1} = D_0$, where D_0 is the initial amount of damage present in the material, we finally get:

$$N_f = N_{th} + \frac{1}{\dfrac{D_0}{D_{cr} - D_0} + \left\{1 - \left[1 - \dfrac{\ln[(p_{th} + \varepsilon_a^p)/p_{th}]}{\ln(\varepsilon_{cr}/\varepsilon_{th})}\right]^{\alpha}\right\}} \tag{23}$$

that corresponds to a linear damage accumulation process. Finally, fatigue life N_f can be directly related to the applied stress amplitude recalling eqns (17) and (18).

Partially coupled life model

A different level of coupling between damage and plastic strain history can be obtained in the estimation of low cycle fatigue life, taking into account that at micro-scale damage affects the material properties. More specifically, it is possible to postulate that, due to the presence of damage, stresses are increased at microscale; consequently, also local plastic strain amplitude, that depends on the accumulated damage, is modified during fatigue life.

Thus, the effect of damage can be taken into account substituting the effective stress in the cyclic Ramberg–Osgood's curve as follows:

$$\varepsilon_a^p = \left[\frac{\sigma_{max}}{(1-D)K'}\right]^{1/n'} \tag{24}$$

where D is evaluated only on the base of the accumulated plastic strain produced during the cycle and is also assumed constant during the single cycle and updated only at the beginning of the new one. Possible modifications of the material constants K' and n' as result of damaging process have been neglected. With these hypotheses, the damage amount per cycle given in eqn (21) cannot be integrated directly, due to the cycle by cycle updating of the effective local plastic strain amplitude, and has to be evaluated numerically.

Fully coupled life model

The number of cycles necessary to activate the damage process has been calculated assuming that each cycle plastic strain accumulation is equal to ε_a^p. It still can be assumed that during damaging cycles, plastic strain continues to accumulate. To take into account the plastic deformation produced during damaging cycles, it can be postulated that in each cycle a new ε_a^p, which is a function of damage, is summed to the previous accumulated plastic strain.

Then, each cycle will see a different plastic strain amplitude and damage state and it will give a different contribution in terms of damage amount produced. In this case, crack

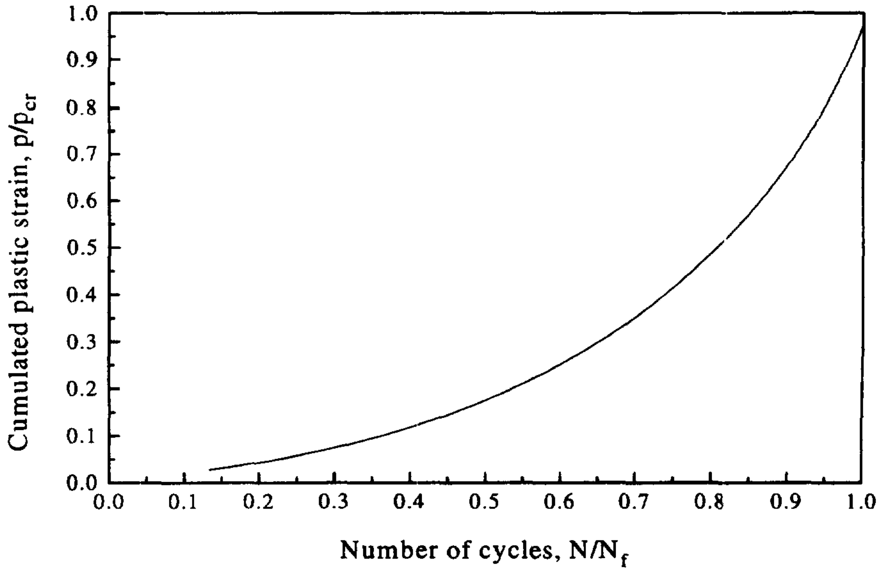

Fig. 3. Example of plastic strain accumulation in a fully coupled fatigue life model.

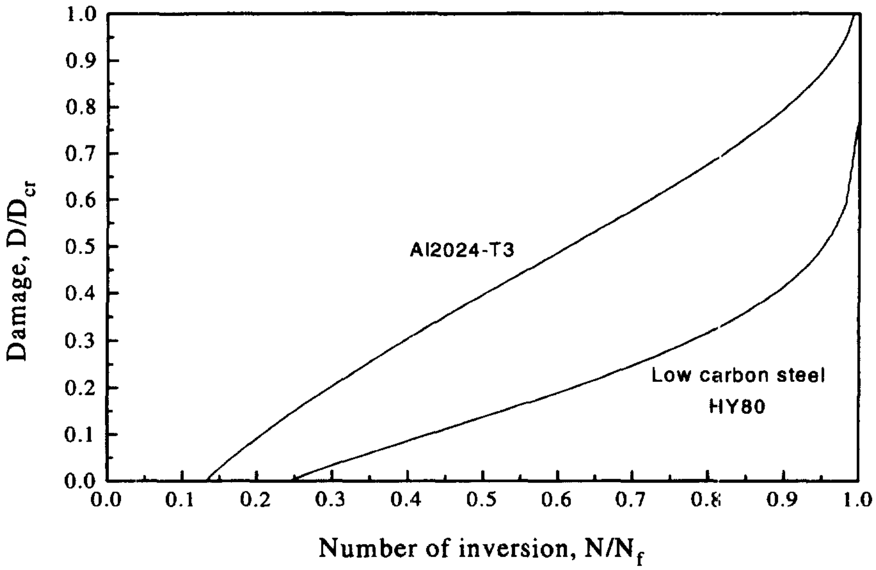

Fig. 4. Different damage accumulation in Al 2024 and HY80 steel, using a fully coupled fatigue life model.

initiation will take place when $D = D_{cr}$ and at the same time p will reach the failure strain p_{cr}. In Fig. 3, an example of the accumulated plastic strain vs the number of cycles is given.

Damage accumulation process turns to be nonlinear according to the damage evolution in the quasi-static case. Differences in the damage variable accumulation is shown in Fig. 4 for Al 2024 alloy and HY80 steel. Finally, fatigue life can be estimated numerically according to the following procedure:

Do Until $D = D_{cr}$

$$\varepsilon_a^p = \left(\frac{\sigma_{max}}{K'(1-D_{i-1})}\right)^{1/n'}$$

$$D_i = D_{i-1} + (D_{cr} - D_{i-1}) \cdot \left\{1 - \left[1 - \left(\frac{D_{cr} - D_0}{D_{cr} - D_{i-1}}\right)^{1/\alpha} \frac{\ln[(p_i + \varepsilon_a^p)/p_i]}{\ln(\varepsilon_{cr}/\varepsilon_{th})}\right]^{\alpha}\right\}$$

$$N_D = N_D + 1 \tag{25}$$

$$D_{i-1} = D_i$$

$$p_i = p_i - \varepsilon_a^p$$

Loop.

These three possible coupling levels between the cumulating variables as damage, plasticity, and material modification as a result of damage, lead to not only a different estimation of the fatigue life, but also show a difference damage variable with number of cycle.

In Fig. 5, the comparison between the different damage accumulation calculated with the three models proposed is given. It is worth underlining, here, that the normalized life was chosen to compare the different damage accumulation, while the number of cycles at failure N_f, estimated with the different models with the same material constants, differs largely for each of the curves.

Even if the uncoupled life model allows us to have a quick, but rough estimation of the fatigue life, the fully coupled life model represents a more consistent framework where damage and plastic strain reach at the same time their respective critical values at the moment of crack initiation. On the basis of the experimental verification illustrated in the next section, it is possible to anticipate that uncoupled and partially coupled models may result in shorter lives. These differences are particularly evident for low stress amplitudes where longer lives are expected. Even if, these estimations are conservative from the safety point of view, they do not allow a rational and effective use of the material in engineering design, underestimating too much the real material endurance.

As a matter of fact, the real mechanism under which a metal fails under repeated loading is not known in detail, yet. The uncoupled and partially coupled low cycle fatigue damage models have been presented to clarify that it is possible to postulate several kinds of interactions between damage and the other cycle variables starting from the simplest

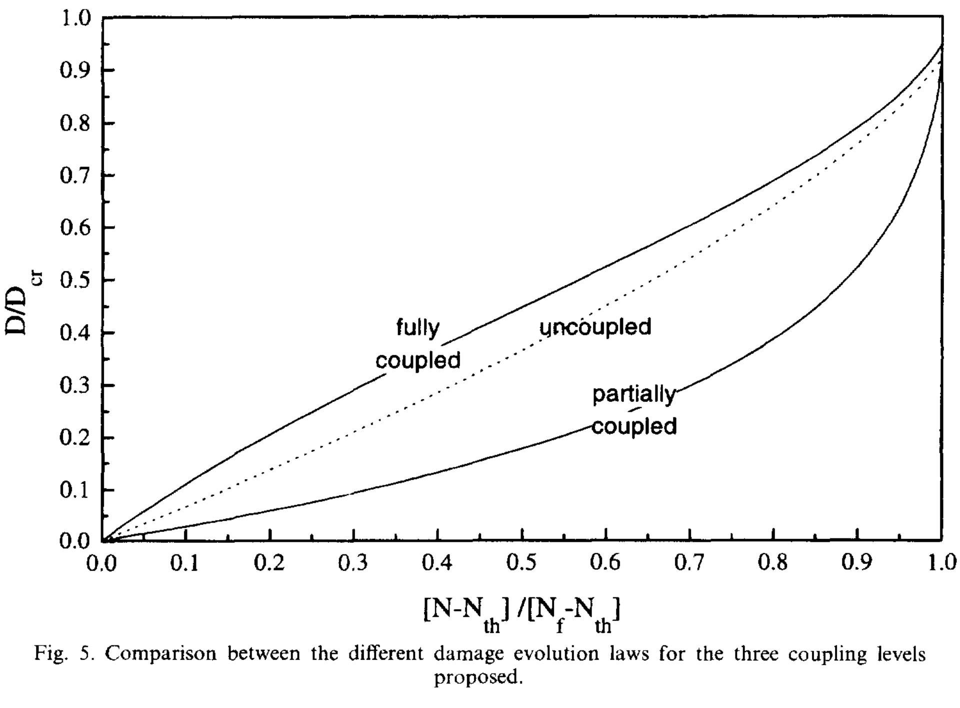

Fig. 5. Comparison between the different damage evolution laws for the three coupling levels proposed.

case. However, the validity of each model, as a result of the hypotheses made, needs to be verified experimentally and only the agreement with the experimental data can give indication on the model complexity.

RESULTS AND DISCUSSION

The low cycle fatigue model proposed has been developed using continuum damage mechanics concepts on the assumption that plastic deformation is the key failure mechanism. In the framework of CDM approach proposed, fatigue life is intended as the number of cycles (or strain reversals) necessary to initiate a macroscopic crack in the material. It does not correspond to the failure life of the specimen or structural component even if, in the case of low cycle fatigue, CDM fatigue life can cover a very large portion of the effective failure life.

The model is able to predict crack initiation in a ductile material on the base of the link between plastic deformation and damage. In order to apply the model, material damage parameters have to be known. Most of the material parameters necessary can be obtained from quasi-static uniaxial damage measurement. There are five materials parameters that need to be measured in this test: the uniaxial failure strain ε_{cr}, damage at failure D_{cr}, the initial amount of damage D_0, the threshold strain ε_{th} for which damage processes are activated, and damage exponent α.

The first four parameters can be measured using several experimental procedures, as described by Lemaitre and Dufailly (1987), but Young's modulus reduction seems to give best results. The damage exponent α can be obtained as the slope of the best linear fit of the damage data on the $\ln[(D_{cr}-D)/(D_{cr}-D_0)]-\ln[\ln(\varepsilon_{cr}/\varepsilon)]$ plane. Detailed description of the specimen geometry, size and experimental procedure can be found in Bonora *et al.* (1994, 1996b). Threshold strain measurements are characterized by a large scatter if performed on uniaxial tensile bars (Hancock and Mackenzie, 1976). Better results can be obtained if threshold strain is measured in triaxial state of stress, as for example, on round notched tensile bar as described in Bonora *et al.* (1996a).

In addition to this, the stabilized cyclic Ramberg–Osgood parameters, K' and n', are necessary to evaluate the effective plastic strain amplitude acting during cycling. The model has been verified for two type of metals that exhibit substantially different damage as a function of strain behavior: an Al 2024-T3 alloy and a low carbon steel HY80. In both cases the fully coupled approach has been used to determine the low cycle fatigue life. In Figs 6 and 7 the comparison between the present model and fatigue experimental data

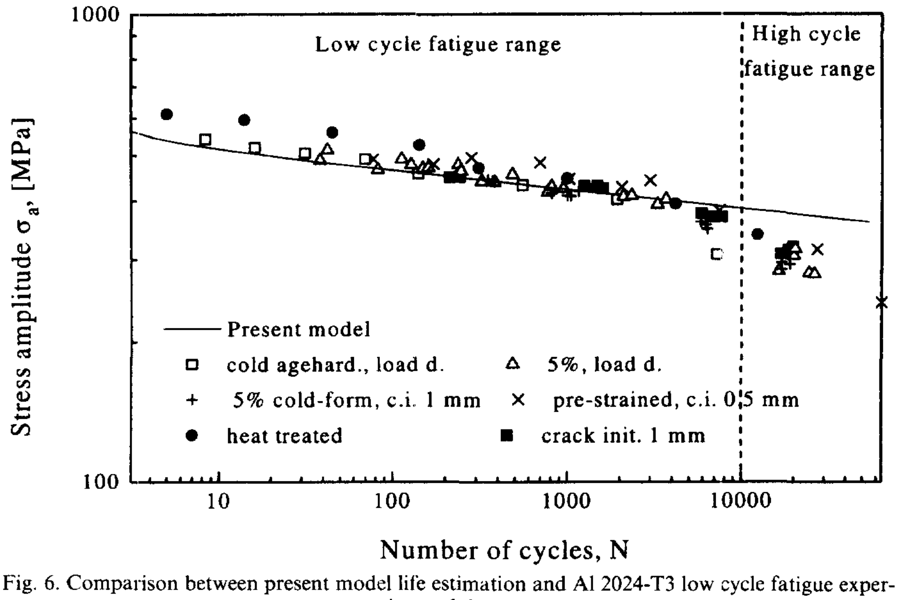

Fig. 6. Comparison between present model life estimation and Al 2024-T3 low cycle fatigue experimental data.

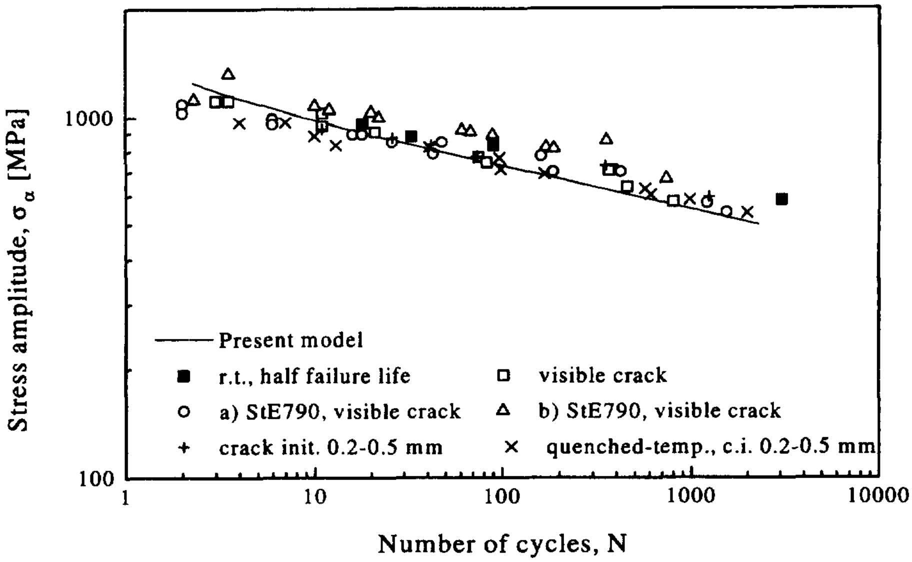

Fig. 7. Comparison between the present model life estimation and HY80 steel low cycle fatigue experimental data.

taken from (Boller and Seeger, 1987a, 1987b) is given. In Table 2, the material cyclic parameters used in this analysis for of the two metals are also summarized.

For each material, several experimental fatigue data sets have been plotted in order to have an idea of the data scatter as results of different material processes as heat treatment, prestraining, etc. In all of the cases, the experimental data presented have been obtained in total strain controlled fatigue tests with cycle ratio $R_\varepsilon = -1$ and each dot on the plots is representative of several measurements. In the experimental tests, the failure number of cycles indicates the number of cycle necessary to generate a macroscopic crack with a length, optically detected, ranging from 0.5 to 2 mm. In some cases, failure data refers to the number of cycles for which the macroscopic specimen stiffness reduction of 5% was observed, that indicates the existence of a macroscopic crack the length of which was not measured. The comparison shows a very good agreement of the model proposed with experimental data and confirms the possibility of approaching low cycle fatigue with CDM.

MEAN STRESS EFFECT

Mean stress or mean strain effects in low cycle fatigue life prediction is still unclear and usually very complex to be taken into account. On the subject several empirical curves obtained from the best fit of experimental data have been proposed (Yao and Munse, 1962; Sessler and Weiss, 1963).

A very simple way to include the mean stress effect in the proposed model is to rewrite the expression of the stress amplitude in terms of the maximum and mean applied stress in order to put in the evidence of the mean stress. Thus

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \sigma_{max} - \sigma_m \tag{26}$$

where σ_m is the mean stress, and σ_{max} is the maximum applied stress both in tension and

Table 2. Material cycle parameters

Material	K' [Mpa]	n'
2024 T3	620	0.4
HY80	1350	0.12

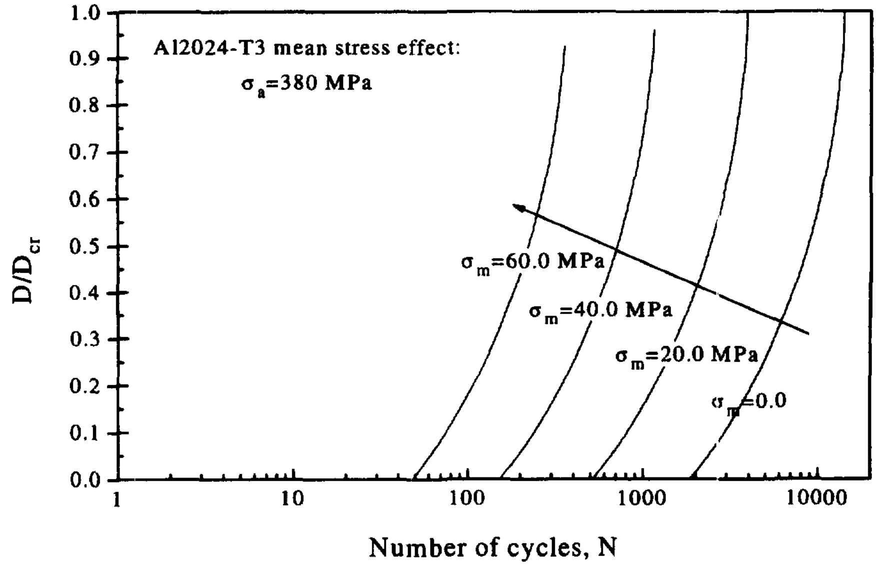

Fig. 8. Mean stress effect on damage evolution law. Plot shows different damage curves for the same stress amplitude and increasing values of mean stress.

compression. If σ_{max} is 0, then the calculated life is infinite and stress needs to be applied in order to induce fracture! Basically it has been observed that the presence of a positive mean stress, with a constant amplitude stress, leads to shorter lives and this is a feature that a fatigue model should include.

Using the definition given in eqn (26), the fully coupled model proposed is able to capture, at least in a qualitative manner, the effect described above. This is shown in Fig. 8, where different damage evolutions are plotted as a function of a number of cycles for a constant stress amplitude and increasing mean stress levels.

CONCLUSIONS

In the present paper a nonlinear continuum damage mechanics model, as proposed by Bonora (1997), has been extended to a cyclic load. The model, that has been shown to be particularly well suited to describe the evolution of ductile damage as a function of strain in metals, is able to predict the effect of cycling loading in the low cycle fatigue range. Three different possible levels of coupling, between cumulated damage, plastic strain and cycles, have been proposed and discussed. The fully coupled life model seems to be more consistent from the conceptual point of view and found a better agreement with the experimental data available in the literature.

The material parameters necessary to use the proposed model can be easily obtained from a quasi-static tensile test and from a step method stable hysteresis loop test in order to determine the cyclic stress–strain curve. Predicted short lives for Al 2024 alloy and HY80 low carbon steel are in good agreement with low cycle fatigue experimental data, which confirms that if the failure mechanism and damage evolution law are known, cyclic material behavior can be directly related to quasi-static damage information.

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